

## [two-columns.tex]

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\begmulti 2 % for two columns,  
\nin      % \nin is needed here  
  {text}  
\endmulti
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N.B. Because of the reduced text width, the nominal font size has been reduced from 12.5 points to 10.5 points with the font command:

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\font\rmtenplus = [/home/liliane/ROGER/MY-FONTS/LinLibertineR.otf] at 10.5 pt
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# The Shape of the Great Pyramid

Roger Herz-Fischler (2000)

## Introduction

Perhaps no other structure built by humans has attracted as much attention as the Great Pyramid of Egypt. Its size, with a base of 230 m and a height of 147 m, is not the sole cause of awe. The setting, on the edge of the desert and overlooking the Nile valley, only adds to its impressiveness, while the complex system of passages, chambers and blockage points, and the yet to be found tomb of the Pharaoh Khufwey (Cheops), have added an aura of mystery. The present work is devoted to what at first glance would appear to be a rather innocent question, “What was the *geometrical*<sup>1</sup> basis, if any, that was used to determine the shape of the Great Pyramid?” However, as the reader can ascertain from its size, there is much more to this book than just giving a mathematical description of a well-known monument from antiquity. In order to better describe its contents, it is necessary to explain its origins.

In 1972 I was asked to teach a mathematics course for first-year students of architecture. Since I was essentially free<sup>2</sup> to choose the topics for the course, I decided to introduce some material dealing with the use of mathematical proportions in architecture. Among the material that I came

across was a statement in Ghyka's 1927 book, *L'Esthétique des proportions dans la nature et dans les arts*, concerning a putative text by the ancient Greek historian Herodotus. This ancient text, it was claimed, explained the shape of the Great Pyramid. Indeed, it seemed from the numbers that were presented by Ghyka, that “theory” and “observation” were in concordance with one another. Not having any reason to doubt what I had read, I presented the theory, over a period of three years, to my classes. It was only later, when I began to write a mathematics textbook for students of architecture,<sup>3</sup> that I tried to locate the quotation by Herodotus. This proved to be impossible, for the putative statement by Herodotus simply did not exist; the only description in the *Histories* of Herodotus which dealt with the dimensions of the Great Pyramid bore little prima facie resemblance to what Ghyka had written.

My curiosity was piqued and thus began a long, tortuous and complicated investigation into the theories that had been proposed concerning the shape of the Great Pyramid. I would come across a new theory and then try to trace it back to its origins, sometimes via comments of others,

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but most often by working backwards through bibliographic references. Thus, what started out as a factual, historical study became a more involved and multi-faceted project. I became interested, not only in the theories and their *history* as such, but also in what I refer to as the *sociological* aspect of these theories; namely how these theories originated, how they were propagated and why some theories survived, whereas others passed into oblivion. This aspect eventually led me back to the Victorian era and to relationships---hardly anticipated at the beginning of my research---between the pyramid theories, and, among other topics, the theory of evolution and the struggle against the introduction of the metric system. Another question also presented itself for, as will be seen, several of the theories gave results which, from a practical viewpoint, were indistinguishable from the observed values. I was thus led to consider *philosophical* questions related to the acceptance of theories.

The present work is the result of my research and reflection. My basic approach in this book is the same as that in my *A Mathematical History of Division in Extreme and Mean Ratio*, my articles in art and architectural history and my forthcoming *The Golden Number*, i.e., keep reading and backtracking through the literature, be skeptical of secondary sources, go off on interesting side tracks,<sup>4</sup> and avoid all preconceived theoretical “approaches” to the subject matter. Above all I believe in letting the material that one finds shape the book rather than writing a book that shapes the material.<sup>5</sup>

The book is divided in three parts which correspond in general terms to the historical and physical background to the theories, the theories themselves, and an overview.

In Part 1, Chapter 1 provides the historical and contextual background for the book. I have summarized, while at the same time giving refer-

ences for those readers who wish to read more detailed discussions, the early history of Egypt and the development of the pyramid. Appendix 1 provides a further, annotated, bibliography of various topics related to the pyramids. Appendix 2 provides a table, together with references, of the dimensions of early pyramids and other tomb superstructures. To my knowledge the set of references to writings on the dimensions and angles of the pyramids is the most complete one available. Chapter 2 begins with the surveyed dimensions of the Great Pyramid and the estimated original angle of inclination of the triangular sides. This is followed by brief discussions of how the Egyptians measured, what their units of measurements were, and what is known of their building techniques. Appendix 3 provides more detailed information on Egyptian units of measure. Chapter 3 is historiographical in nature, and considers previous studies of the theories of the shape of the Great Pyramid.

The second part of the book begins with diagrams which illustrate the different ways in which the shape of a pyramid can be defined and gives the terminology employed in the rest of the book. Part 2 begins in Chapter 4 with a comparative table of the theories and the angles of inclinations of the faces which correspond to these theories. I also point out parallels between certain of the theories. Then follows, in Chapters 5 through 15, the historical and sociological developments of the eleven theories that are known to me. The presentation is in chronological order, with respect to the first known appearance of the theory. The one exception is the *seked* theory of Chapter 5, for which the theoretical basis is an ancient Egyptian text. I thus presented this theory first, even though a formal connection with the Great Pyramid was not stated until 1922.

Each chapter begins with a brief mathematical description, in simple trigonometric or geometric language, of the theory in question. The first

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note of each section contains a complete list of the angles, lengths etc. associated with this theory. The formulae for computing these quantities are given in the notes to Chapter 4. The rest of each chapter is then a mixture of historical and sociological material, including a description of the mathematical approach of different authors.

Several of the Chapters in Part 2 contain special material, which I felt was necessary for a proper understanding of the background of the theory. Chapter 5 includes archaeological evidence related to the *seked* theory, as well as a discussion of the pyramid problems in the *Rhind Papyrus*. Similarly Chapter 9 discusses what the Egyptians knew about circle calculations. Other aspects of Egyptian mathematics are summarized in Appendix 4. The text of Herodotus cited above in connection with the book by Ghyka, and which constitutes the “historical” basis for two of the theories, is discussed in Chapter 6, with Appendix 5 providing a technical background for Greek and Greek-Egyptian systems of measures. Chapter 7 contains a discussion of another ancient text which has formed the theoretical basis for the discussions of various authors, namely Plutarch's *Isis and Osiris* in which the 3--4--5 triangle is related to these Egyptian gods. Chapter 16 presents some additional material which, while never appearing as formal theories of the shape of the Great Pyramid, is of interest in the context of this book.

Part 3 begins with a discussion of *philosophical* matters related to the theories. One notes immediately that there are only very small differences between the angles resulting from the theories and the observed value of the angle of inclination of the faces. Since the correct theory cannot be determined on the basis of numerical accuracy---or to look at the matter in another way, cannot be rejected on the basis of a discrepancy between theory and observation---

philosophical questions arise as to when we can, or should, accept or reject a theory. Chapter 17 proposes some criteria related to the acceptance of theories.

Chapter 18 is devoted to a case study of the *sociology* of the pi-theory. As we shall see, the pi-theory is a true theory of Victorian Britain and so we have a very special opportunity to observe the conditions which give rise to a theory and cause it to be widely disseminated. The first section of Chapter 18 discusses the social and intellectual background in Victorian Britain which gave rise to the pi-theory and led to its widespread dissemination. The next section deals with the four topics of great interest in that period with which the pi-theory was associated: the “squaring of the circle” , units of measure, the Bible, and the theory of evolution. The last section deals with the authors themselves. By means of specialized biographical sources, I have made an analysis of the background, occupation and interests of the nine principal Victorian authors who wrote on the pi-theory. I hope that the reader will find the maze of interconnected external influences and people as fascinating as I did.

Chapter 19 contains my conclusions. The first section deals with my observations as to how theories propagate and in particular why certain theories flourished whereas others essentially disappeared. The second section of Chapter 19 returns to the question, “What was the geometrical basis that was used to determine the shape of the Great Pyramid?”.

The bibliography contains some 315 items. Since many of the primary and secondary sources are very difficult to locate or obtain, I have indicated with each bibliographic entry, except for very common twentieth-century material, the library that was kind enough to lend me the material. For certain bibliographical entries, I have added

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comments or references to other works so as to aid future researchers. Since this is to a large extent a book about books and articles, I felt that it would be more useful to the reader to have an index to an author's individual books rather than just having an index with only the names

of the authors. Thus the bibliography also serves as the index, with the location of the discussion of a book or article being given at the end of the bibliographic entry. The detailed table of contents provides another entry to the authors and topics discussed.

